Effect of environment on the interferometry of clocks

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THE UNIVERSITY OF QUEENSLAND AUSTRALIA

CREATE CHANGE

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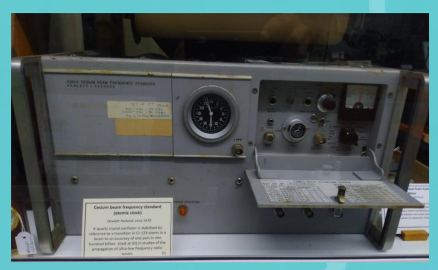
INTRODUCTION

- Objective: To probe genuine relativistic effects in quantum systems.
- Proposal: Using internal, dynamical degree of freedom (DOF) as a ***clock*** to capture time dilation[1]. Here, we have a two-level system as a clock with the Hamiltonian - $\mathbf{H_0}$.
- For example: Energy levels of an atom, vibrational levels of a molecule etc.
- Problem: In an interferometric setup, the internal DOF is susceptible to environmental effects/ noise.



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Quantum superposition at the half-metre scale

T. Kovachy, P. Asenbaum, C. Overstreet, C. A. Donnelly, S. M. Dickerson, A. Sugarbaker, J. M. Hogan & M. A. Kasevich

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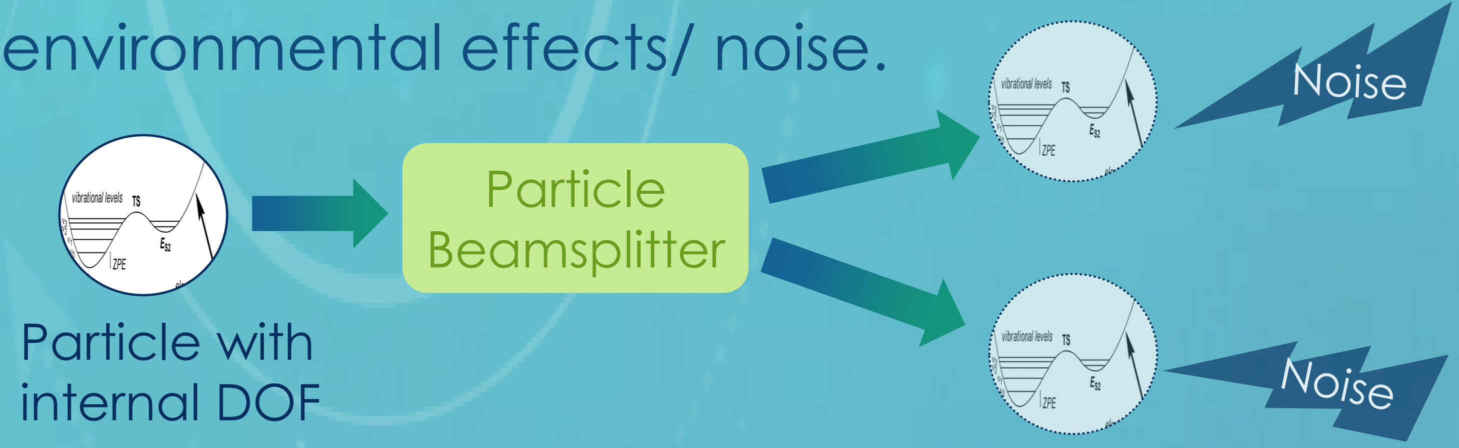
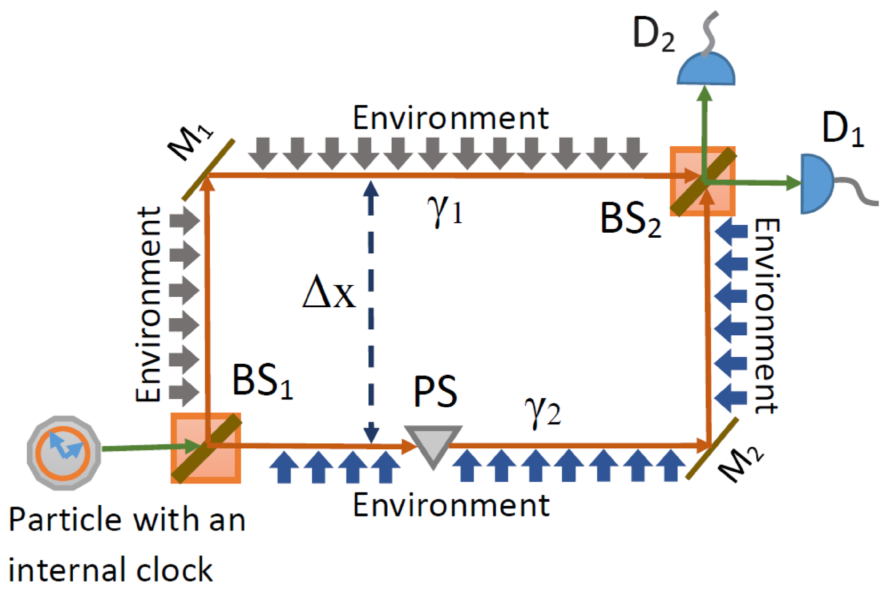


MZ INTERFEROMETER



- BS_1 / BS_2 : Beamsplitters
- PS : Phase shifter
- γ_1 / γ_2 : Paths - |1> & |2>
- D_1 / D_2 : Detectors
- M_1 / M_2 : Mirrors

- Initialization of clock: $|\psi_{\rm clock}\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$
- Evolution in proper time: $|\tau\rangle := e^{-iH_{\rm int}\tau}\,|\psi_{\rm clock}\rangle$
- Entanglement of the clock with the path degree of freedom of the particle by virtue of time dilation[1]:

$$|\psi_{\rm out}\rangle = \frac{1}{\sqrt{2}}[|1\rangle\,|\tau_1\rangle + e^{i\chi}\,|2\rangle\,|\tau_2\rangle]$$

- Interferometric Visibility: $\mathcal{V} = |\langle\tau_1|\tau_2\rangle|$
- Visibility depends on the difference in proper time (Δτ):

$$\mathcal{V} = |\,\langle\psi_{\rm clock}|\,e^{-iH_{\rm int}\Delta\tau}\,|\psi_{\rm clock}\rangle\,|$$

- Complementarity relation[2]:

$$\mathcal{V}^2 + \mathcal{D}^2 \leq 1$$

ENVIRONMENT

- Model: Extra DOFs interacting with the clock system[3].
- Contradictory intuitions due to environment:
  - Corruption of which-path information $\Longrightarrow \mathcal{D}\downarrow \Longrightarrow \mathcal{V}\uparrow$
  - Larger Hilbert space of clock system $\Longrightarrow \mathcal{D}\uparrow \Longrightarrow \mathcal{V}\downarrow$
- Intuitive regime: For low noise, small Δτ, $\mathcal{V}\downarrow$

VISIBILITY WITH NOISY CLOCKS

- $U_{\rm noise}$ for the extended system can be known for standard noise models – amplitude damping (AD), phase damping (PD) and depolarizing (DP) channels.
- Using eigendecomposition, we get the corresponding Hamiltonian- $H_{\rm noise}$. The total Hamiltonian is:

$$H_{\rm int} = H_0 + H_{\rm noise} + H_{\rm env}$$

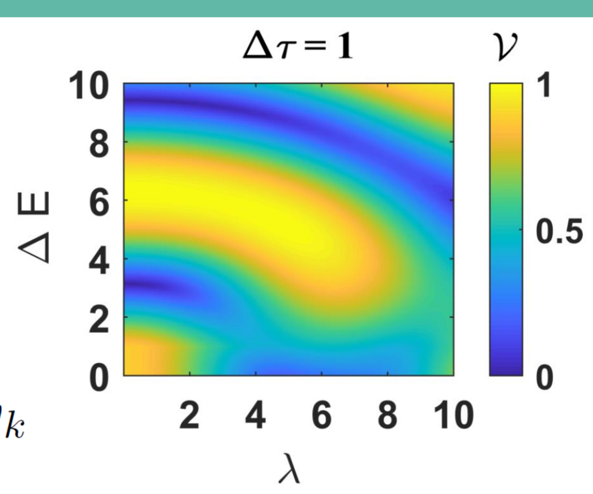
JAYNES CUMMINGS MODEL

- Single bosonic field mode.

$$H_{\rm int} = \underbrace{\frac{\Delta E \sigma_C^z}{2}}_{H_0} + \underbrace{\omega_k a^\dagger a}_{H_{\rm env}} + \underbrace{\frac{\lambda}{2}(a\sigma_C^+ + a^\dagger\sigma_C^-)}_{H_{\rm noise}}$$

$$\alpha_0 = \tan^{-1}\left(\frac{\lambda}{\delta}\right),\ \lambda_0 = \sqrt{\delta^2+\lambda^2},\ \delta = \Delta E - \omega_k$$

$$\mathcal{V} = \frac{1}{2}\left|e^{i\frac{\Delta E}{2}\Delta\tau} + e^{-i\frac{\omega_k+\lambda_0}{2}\Delta\tau}\cos\left(\frac{\alpha_0}{2}\right)^2 + e^{-i\frac{\omega_k-\lambda_0}{2}\Delta\tau}\sin\left(\frac{\alpha_0}{2}\right)^2\right|$$

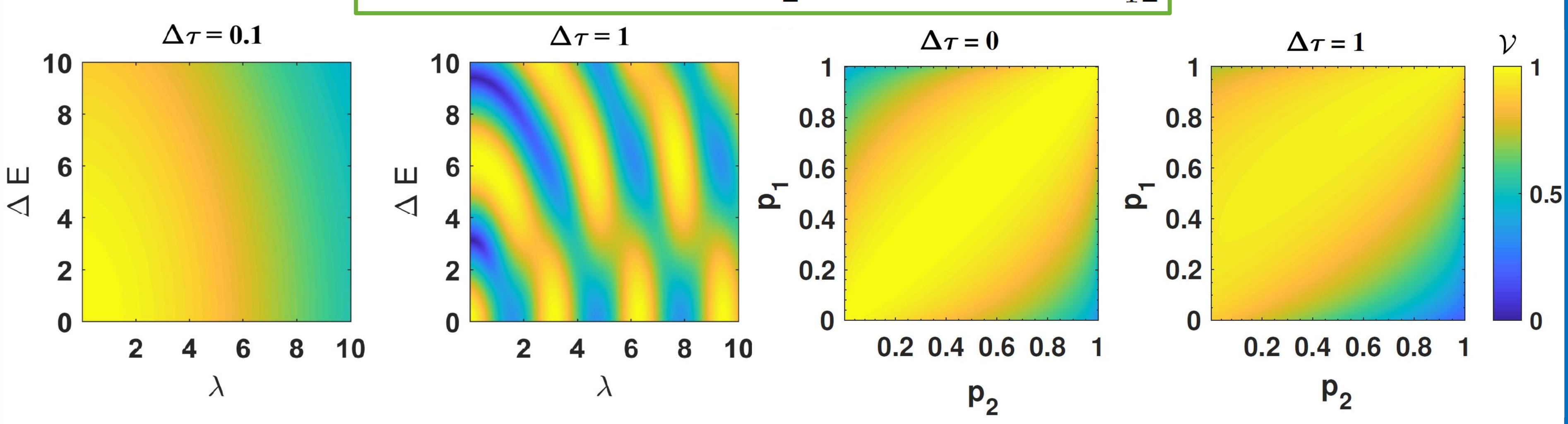


AMPLITUDE DAMPING CHANNEL

$$U_{\rm AD}(\tau^*) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{1-p} & \sqrt{p} & 0 \\ 0 & -\sqrt{p} & \sqrt{1-p} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow[\lambda = \frac{\theta}{\tau^*}]{p=\sin^2\theta} \begin{matrix} H_{\rm AD} = -\frac{\theta}{\tau^*}\,|\psi_1\rangle\langle\psi_1| + \frac{\theta}{\tau^*}\,|\psi_2\rangle\langle\psi_2| \\ \equiv 2i\lambda\left[|10\rangle\langle 01| - |01\rangle\langle 10|\right] \end{matrix}$$

$$H_0 = (E_0|0\rangle\langle 0| + E_1|1\rangle\langle 1|)\otimes \mathbb{1} = E_0|00\rangle\langle 00| + E_0|01\rangle\langle 01| + E_1|10\rangle\langle 10| + E_1|11\rangle\langle 11|$$

$$H_{\rm int} = H_0 + H_{\rm AD} = \begin{bmatrix} E_0 & 0 & 0 & 0 \\ 0 & E_0 & -2i\lambda & 0 \\ 0 & 2i\lambda & E_1 & 0 \\ 0 & 0 & 0 & E_1 \end{bmatrix}$$

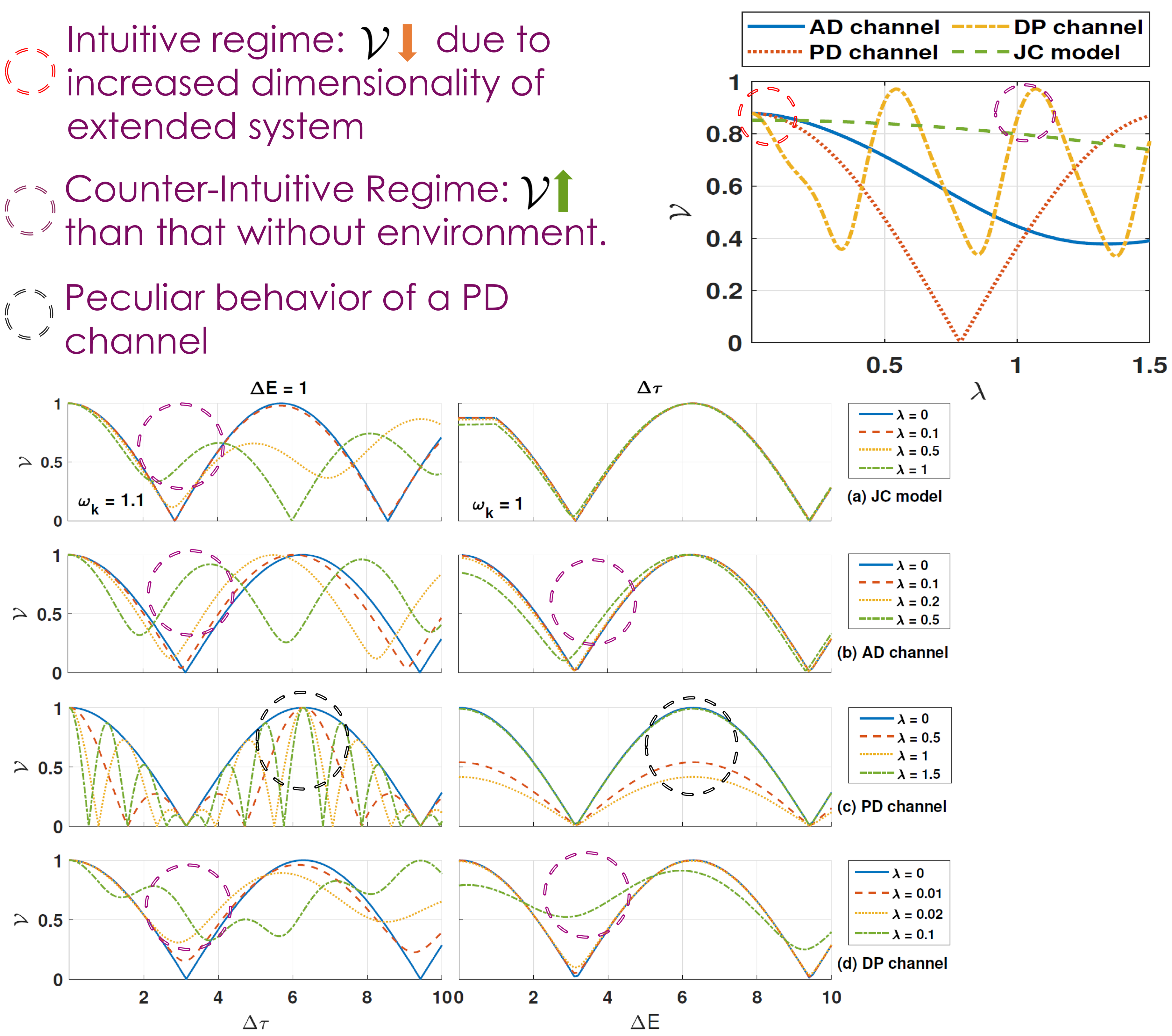


Interplay between clock ($\Delta E = E_1 - E_0$) and environment (λ): $\lambda_1 = \lambda_2$

Different environments (λ) along the paths : $\lambda_1 \neq \lambda_2$

COMPARISON BETWEEN MODELS

- Intuitive regime: $\mathcal{V}\downarrow$ due to increased dimensionality of extended system
- Counter-Intuitive Regime: $\mathcal{V}\uparrow$ than that without environment.
- Peculiar behavior of a PD channel



KEY POINTS

- Noise on internal DOF affects the Visibility.
- Locally acting environment also subscribes to time dilation.
- Visibility may also be more than that without the environment for AD and DP channel based noise models.
- For different types of environments, visibility varies in susceptibility to λ – the noise parameter.
- Important implications for actual experiment.

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