Energetics of VQE for Heisenberg model

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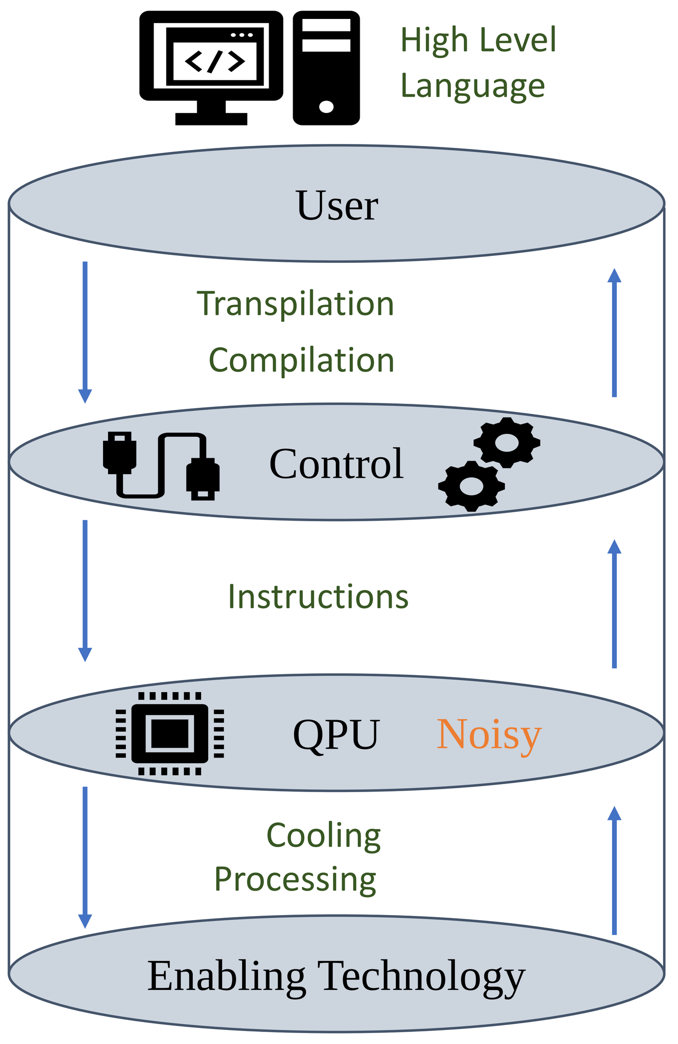
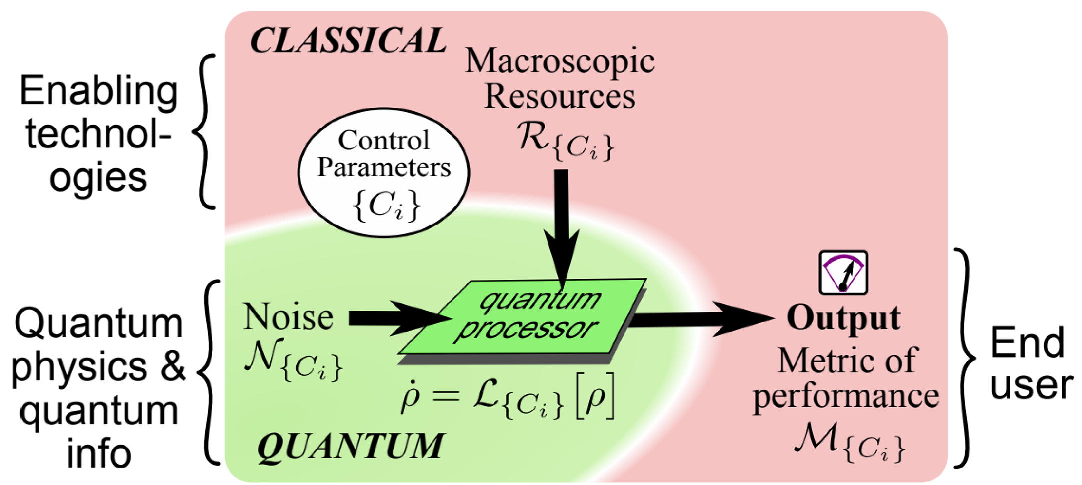
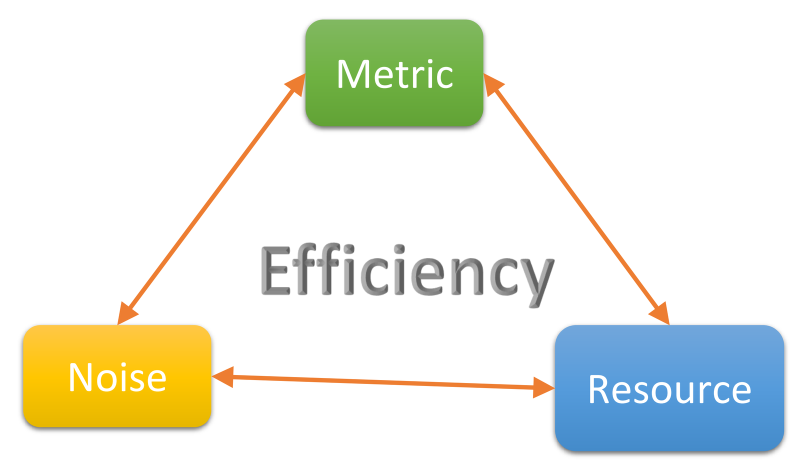
M-N-R framework[6]







Level	Metric	Parameters affecting the metric and resource
Component (Gate)	Gate fidelity	Enabling tech, qubit quality, control system
System (Circuit)	Circuit fidelity, success rate	All of the above, Compilation, transpilation
User (Algorithm)	Ground state energy of molecule	All of the above, choice of algorithm, post-processing

VQE for Heisenberg Model

Heisenberg Hamiltonian $$H = \sum_{i>j}^{N} \left(J_{ij}^{xx} \sigma_i^x \cdot \sigma_j^x + J_{ij}^{yy} \sigma_i^y \cdot \sigma_j^y + J_{ij}^{zz} \sigma_i^z \cdot \sigma_j^z \right)$$

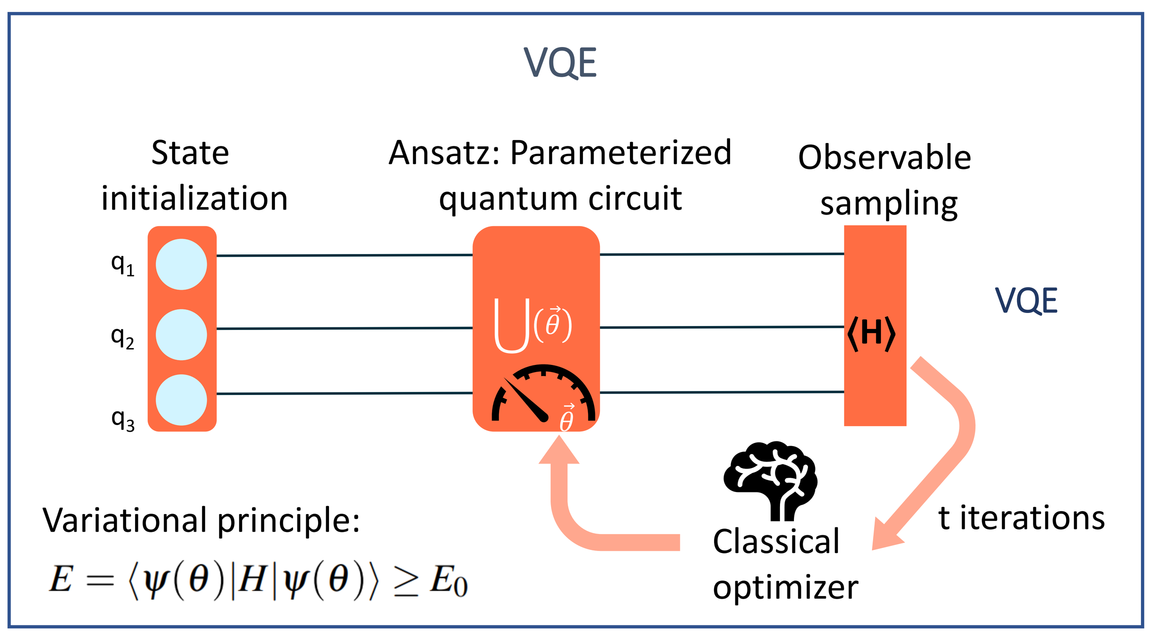
MNR

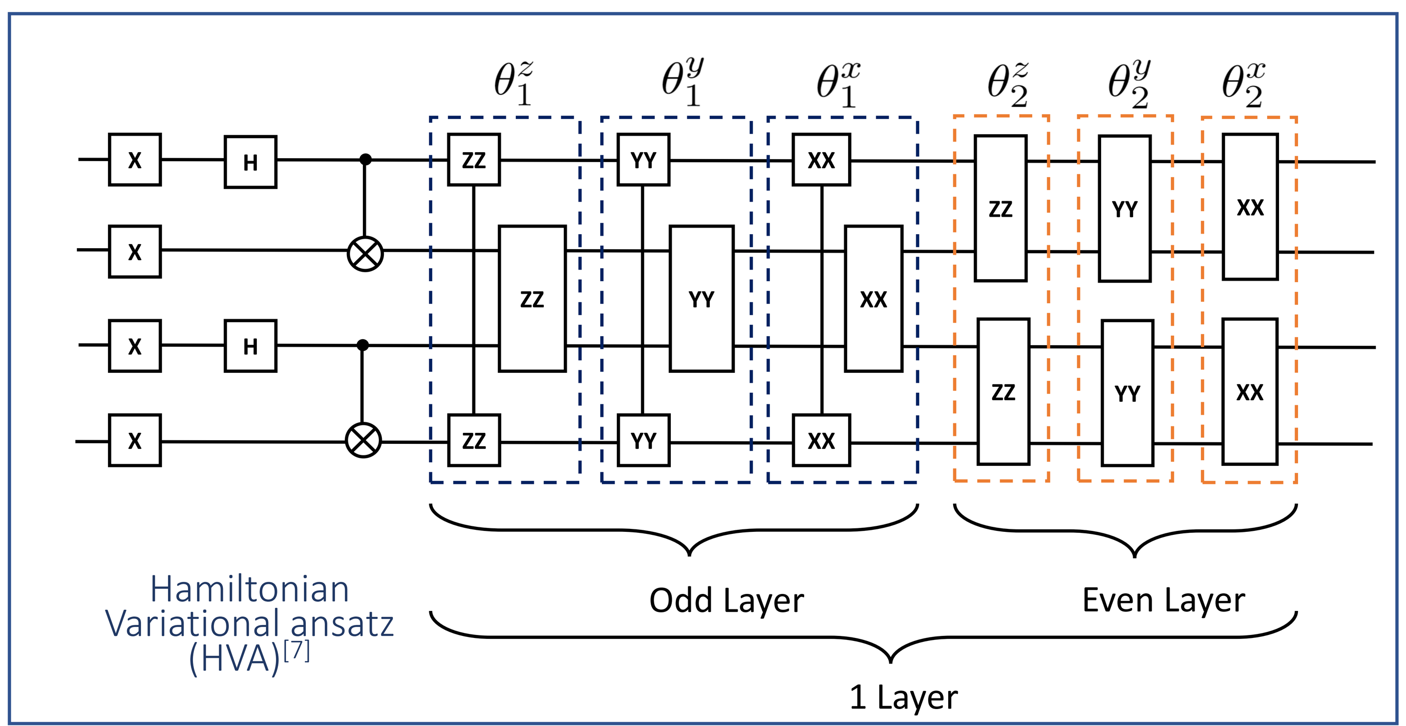
Resources

- Number of VQE iterations : t
- Number of gates : N_g
- Number of measurements

Metric

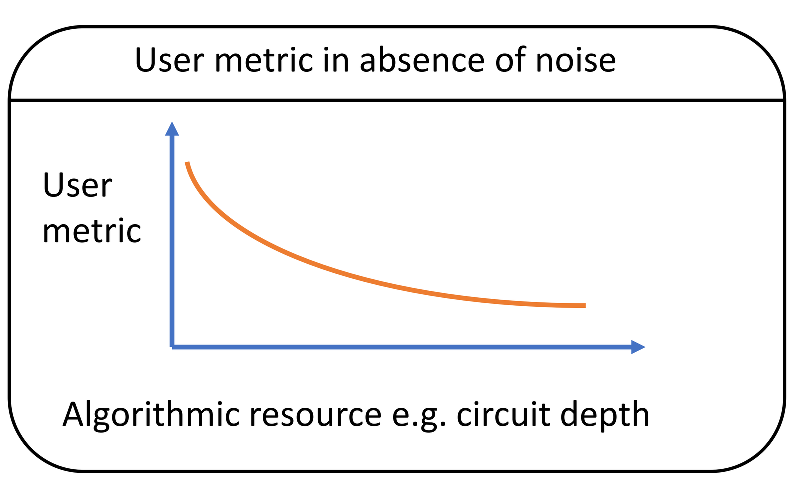
- Accuracy of the calculated ground state energy

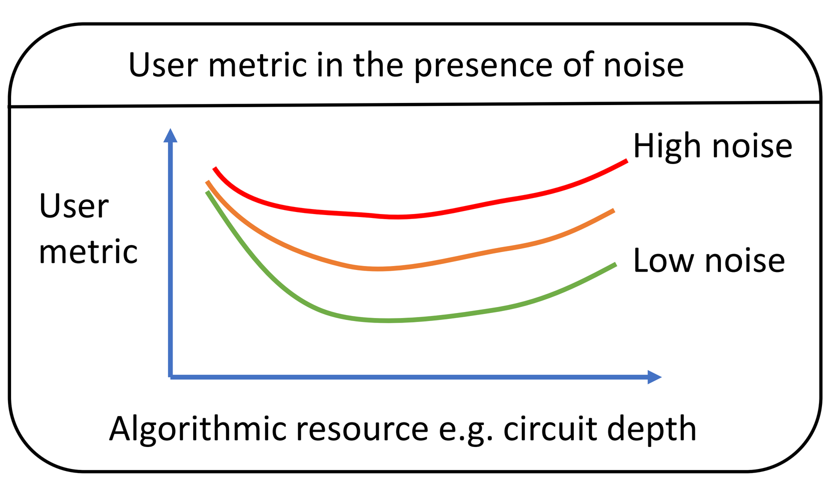




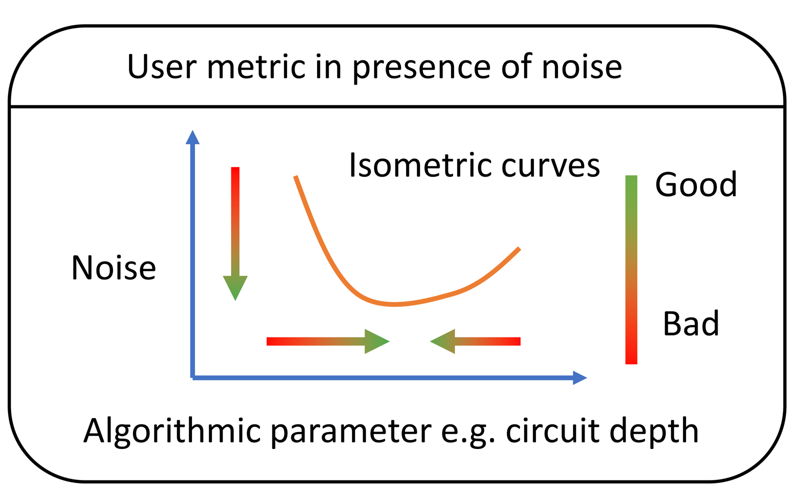
Hardware Agnostic MNR in VQE

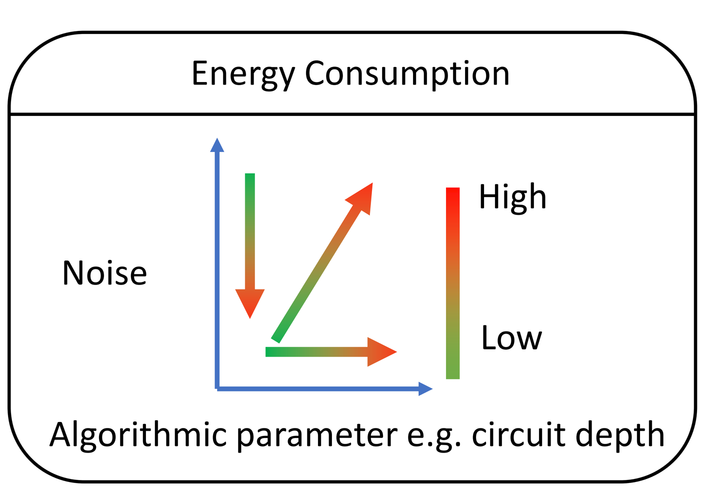
In VQEs, the accuracy of results increases with number of 'layers' of the variational circuit.





Characterizing efficiency using user metric and energy consumption in the MNR framework[1]





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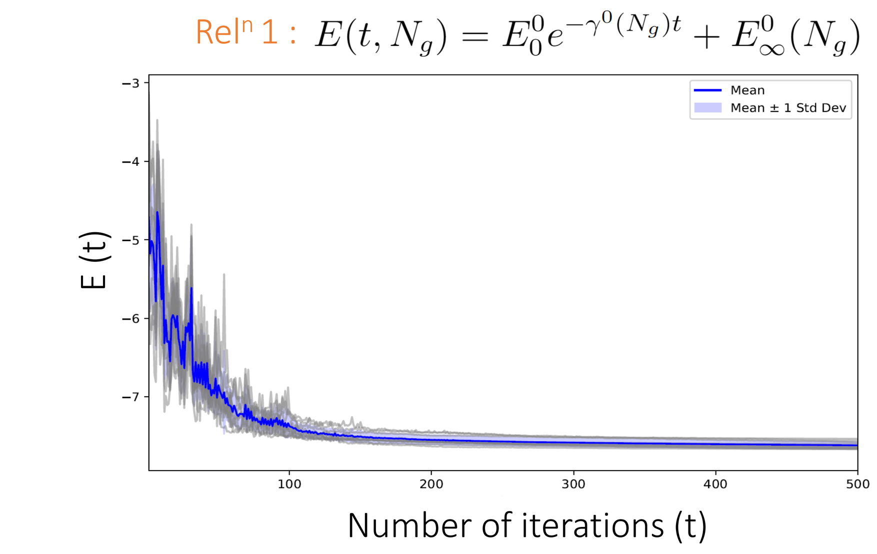
Phenomenological Model for VQE

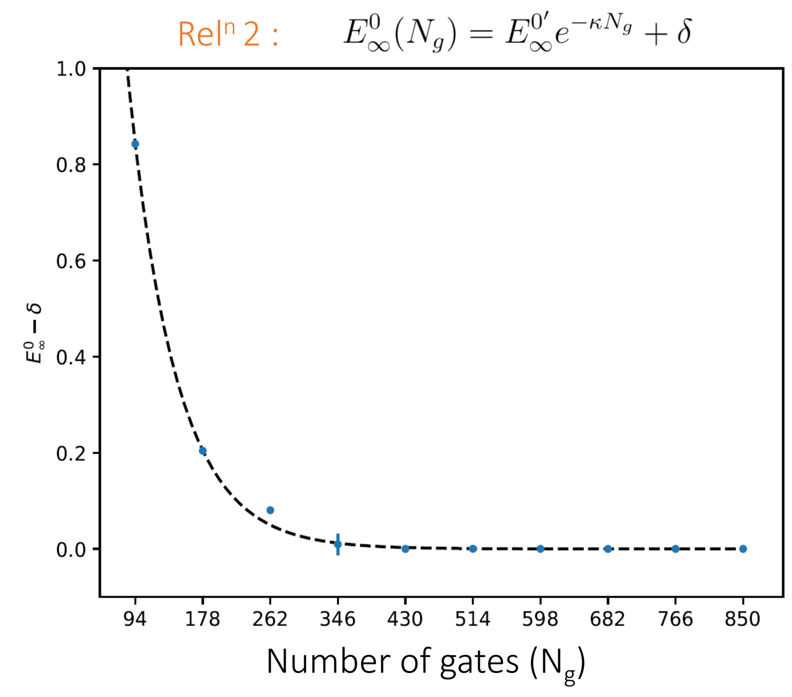
Noiseless convergence with number of iterations

Reln 1 : $E(t, N_g) = E_0^0 e^{-\gamma^0(N_g)t} + E_\infty^0(N_g)$



Noiseless convergence with number of Gates

Reln 2 : $E_\infty^0(N_g) = E_\infty^{0'} e^{-\kappa N_g} + \delta$



Global depolarizing Noise Model

$$\tilde{\rho} = \left[(1-\epsilon)^{\mathcal{N}}\right] U(\theta)\rho U(\theta)^\dagger + \left[1-(1-\epsilon)^{\mathcal{N}}\right]\frac{\mathbb{I}}{2^n}$$

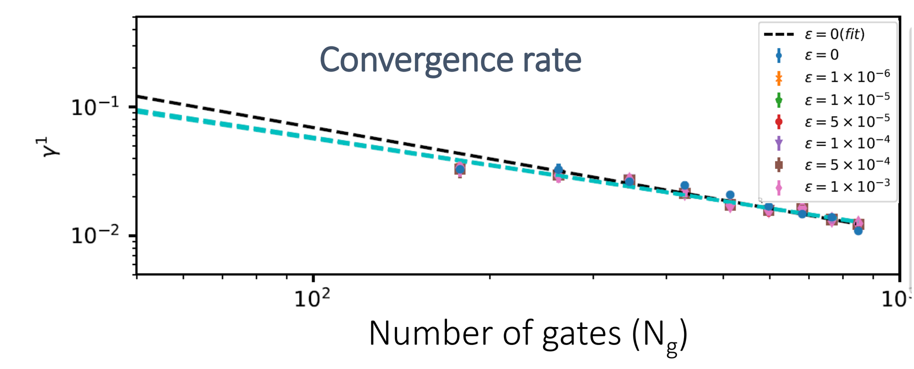
$$E_{noisy}(\theta) = \left[(1-\epsilon)^{\mathcal{N}}\right] \mathrm{Tr}(U(\theta)\rho U(\theta)^\dagger) + \left[1-(1-\epsilon)^{\mathcal{N}}\right]\frac{\mathrm{Tr}(H)}{2^n}$$

Convergence of noisy VQE with iterations (t)

$$E(t, N_g) = \left[(1-\epsilon)^{\mathcal{N}}\right] E_0^0 e^{-\gamma^1(N_g)t} + E_\infty^1$$

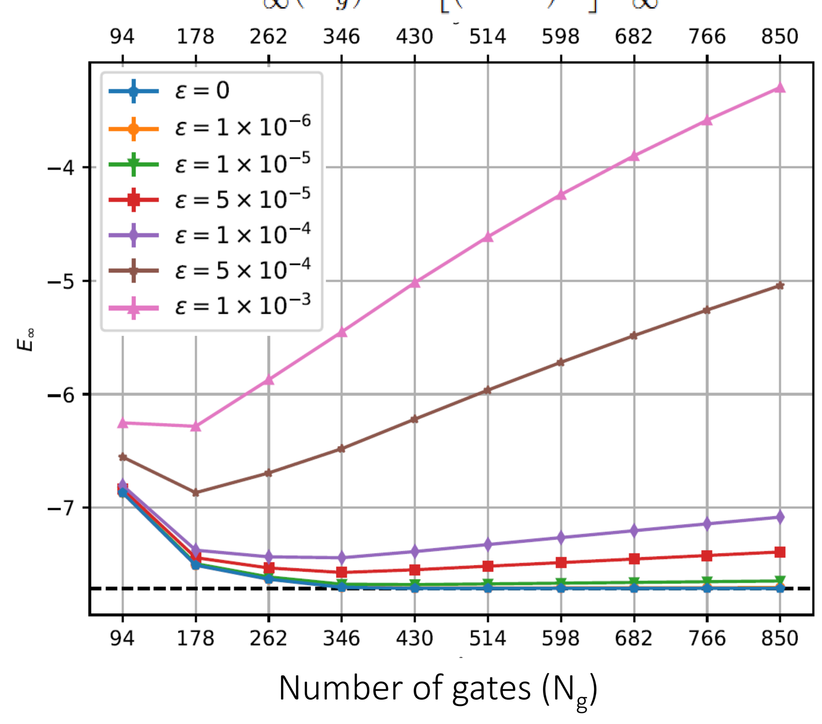
Noisy convergence rate

$$\gamma^1(N_g) = \gamma_1 N_g^{-\omega_1}$$



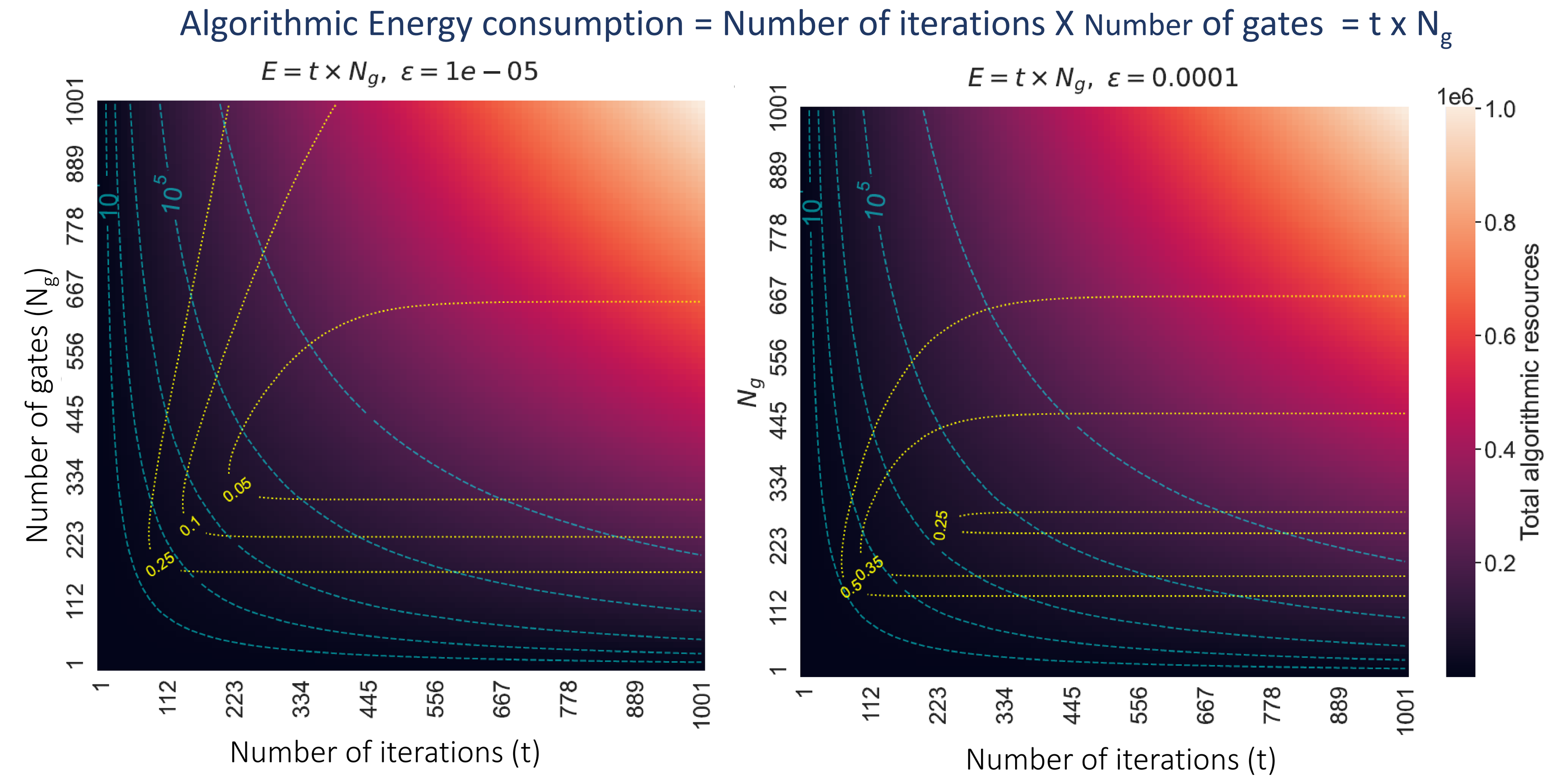
Noisy converged value of ground state energy

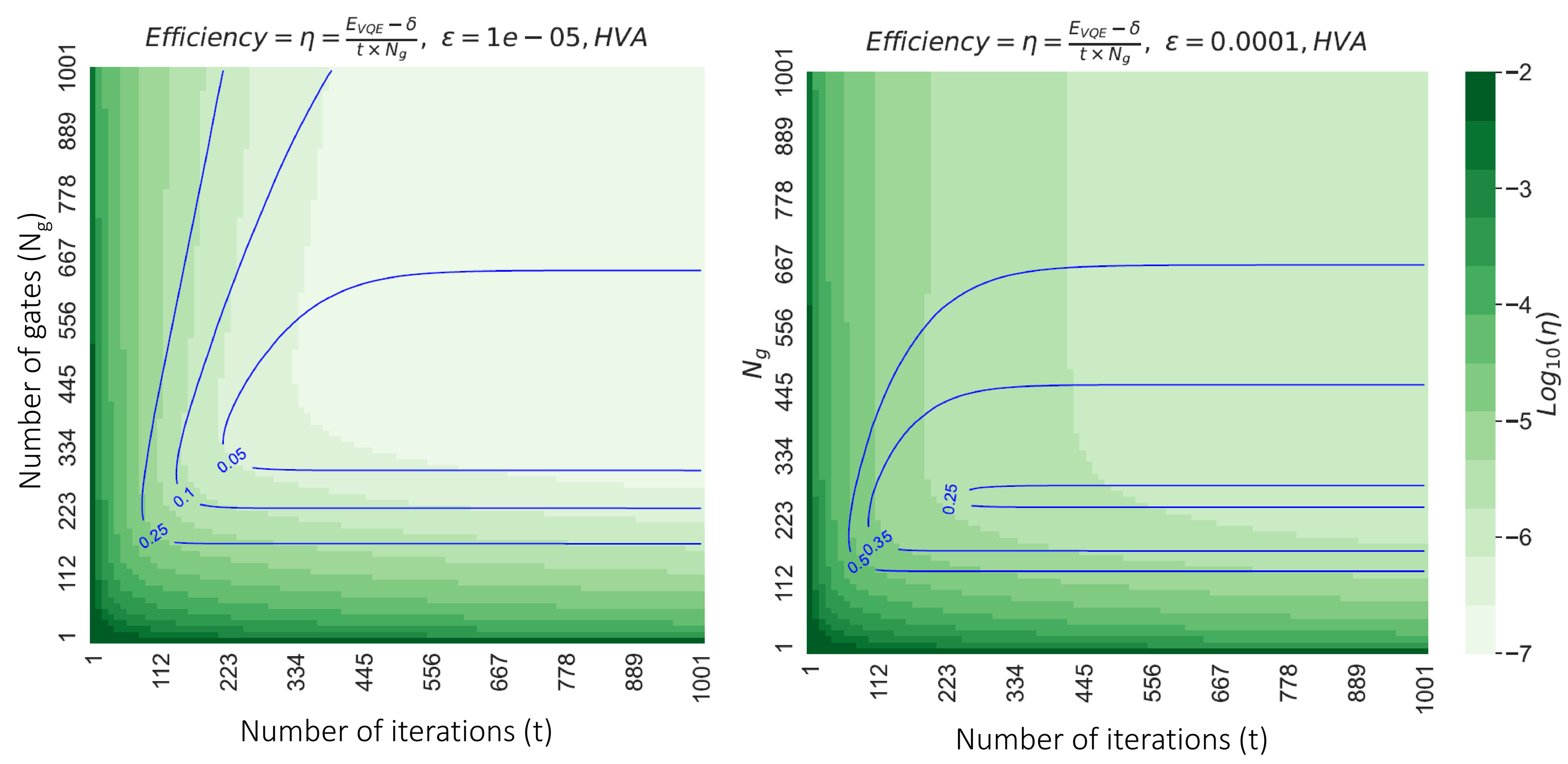
$$E_\infty^1(N_g) = \left[(1-\epsilon)^{\mathcal{N}}\right] E_\infty^0$$



Energetics: Resource tradeoff and efficiency

Algorithmic Energy consumption = Number of iterations X Number of gates = t x N_g





Outlook

- Translate **energy consumption from algorithmic resources to physical energy costs** in the context of hardware specific scenarios.
- Ascertain the **effect of software stack** on the performance and efficiency.
- Development of **heuristic optimizer** to find the minimal resource cost for a given target user metric in a hardware agnostic setting.
- Estimate **classical resource cost** to compare against the quantum case, to identify possible scenarios of **quantum energetic advantage**.
- Deduce the corresponding **hardware parameters for achieving the energetic advantage**.
- Development of an **energetic benchmark** based on applications relevant for many body quantum systems.

References

[6] Marco Fellous-Asiani, Jing Hao Chai, Yvain Thonnart, Hui Khoon Ng, Robert S. Whitney, and Alexia Auffèves *Optimizing resource efficiencies for scalable full-stack quantum computers*. PRX Quantum **4**, 040319 (2023).

[7] Wiersema, R., Zhou, C., de Sereville, Y., Carrasquilla, J. F., Kim, Y. B., & Yuen, H. *Exploring Entanglement and Optimization within the Hamiltonian Variational Ansatz*. PRX Quantum, 1(2), 020319 (2020).