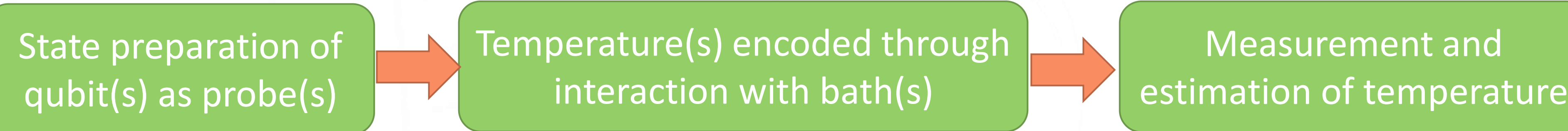


THERMOMETRY AND BOUNDS ON ESTIMATION

Measurement & estimation scheme:



Multiparameter CR bound :

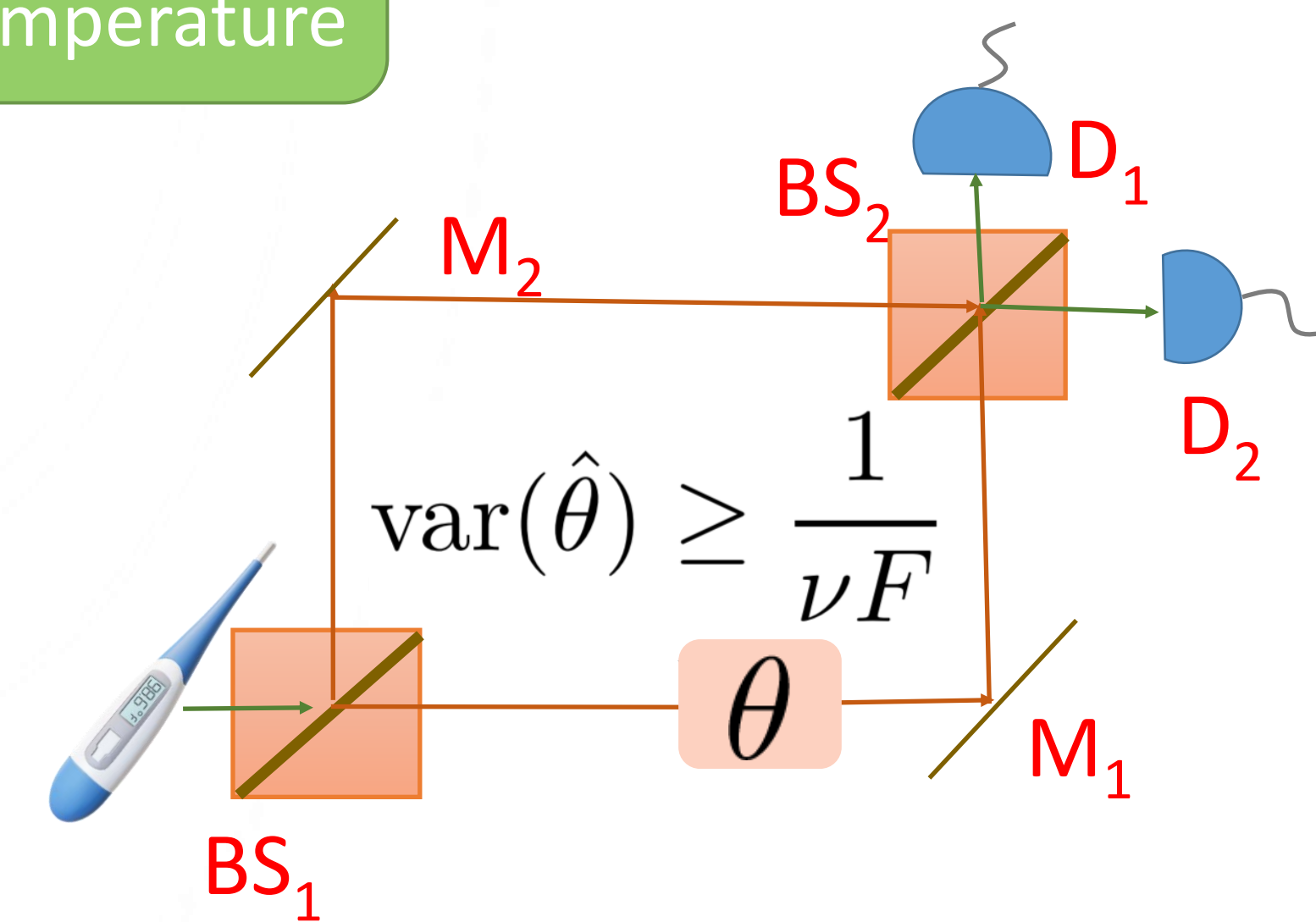
$$\text{Cov}(\vec{T}) \geq \frac{Q_T^{-1}}{\nu}$$

Bounds on variances obtained from the above matrix inequality :

$$\text{Var}(T_1) \geq \frac{Q_{T_2 T_2}}{\nu \det(Q_T)}$$

$$\text{Var}(T_2) \geq \frac{Q_{T_1 T_1}}{\nu \det(Q_T)}$$

$$\left(\text{Var}(T_1) - \frac{Q_{T_2 T_2}}{\nu \det(Q_T)} \right) \left(\text{Var}(T_2) - \frac{Q_{T_1 T_1}}{\nu \det(Q_T)} \right) \geq \left[\text{Cov}(T_1, T_2) + \frac{Q_{T_1 T_2}}{\nu \det(Q_T)} \right]^2$$

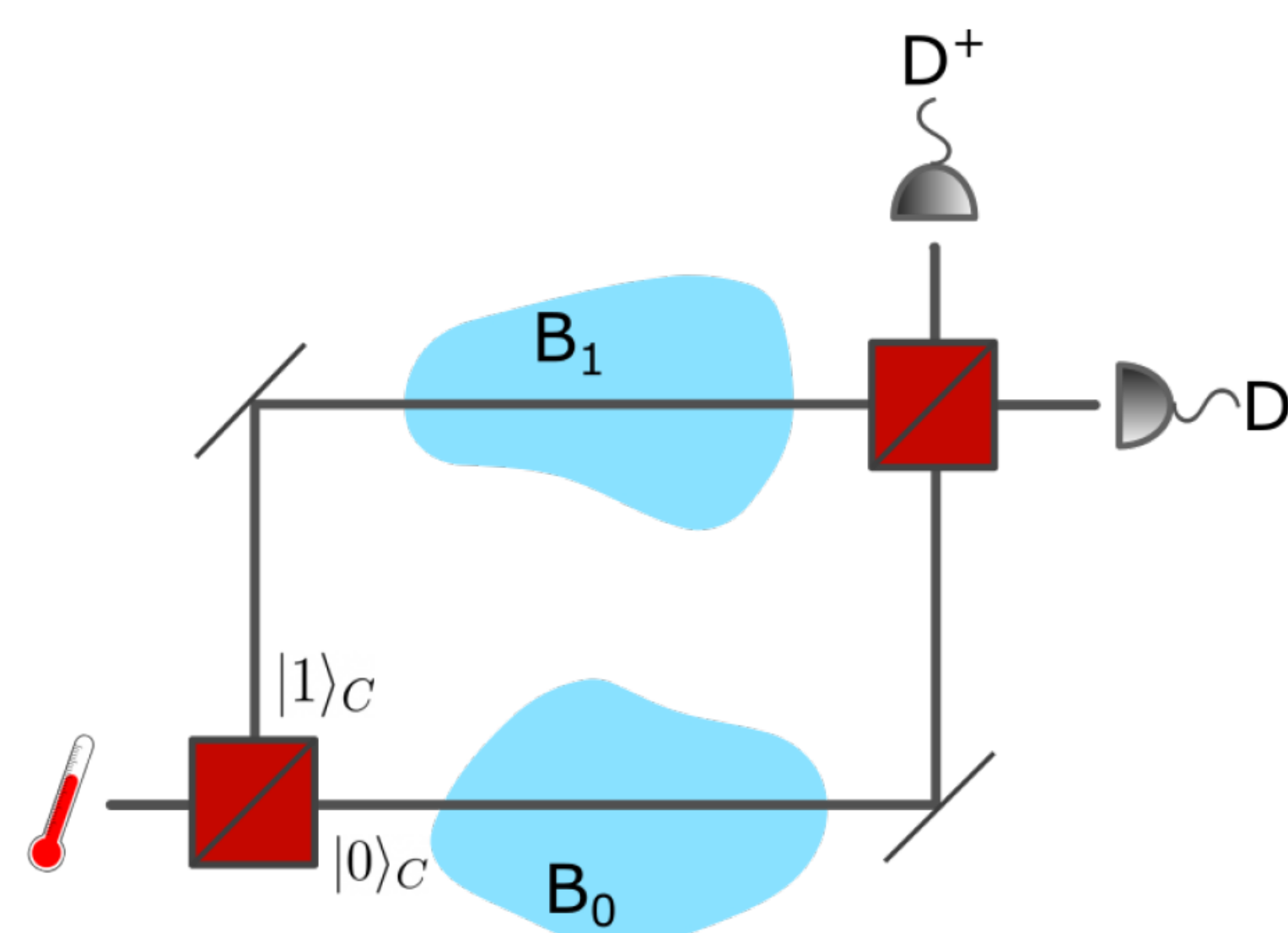


ν - Repetitions of expt.
 F - Q. Fisher Information

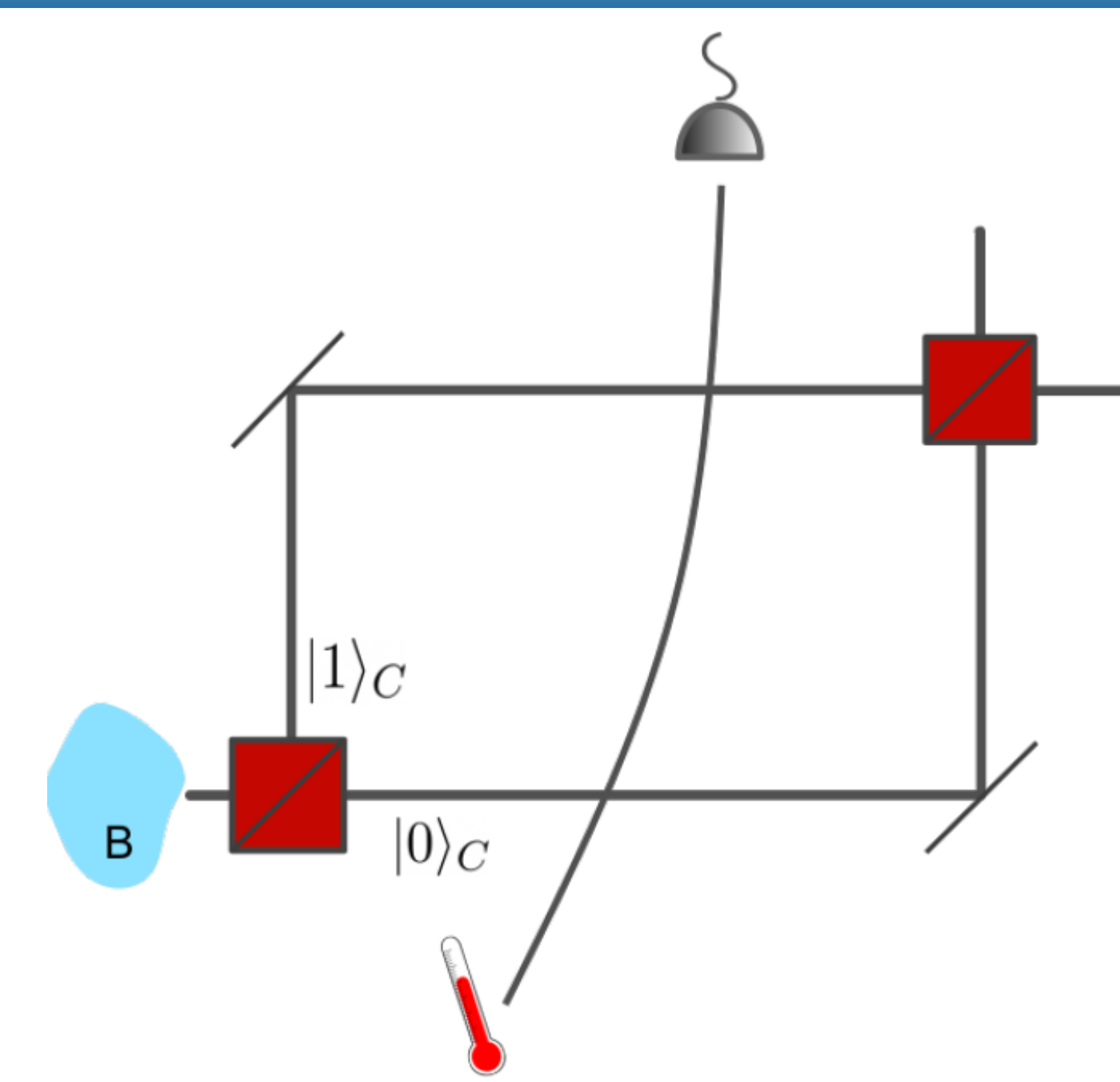
OPERATIONAL MODELS OF SUPERPOSITION OF TEMPERATURES

- Question: What does it mean to have a superposition of thermalizing quantum channels ?
- Approaches^[1]:

2 - bath case:



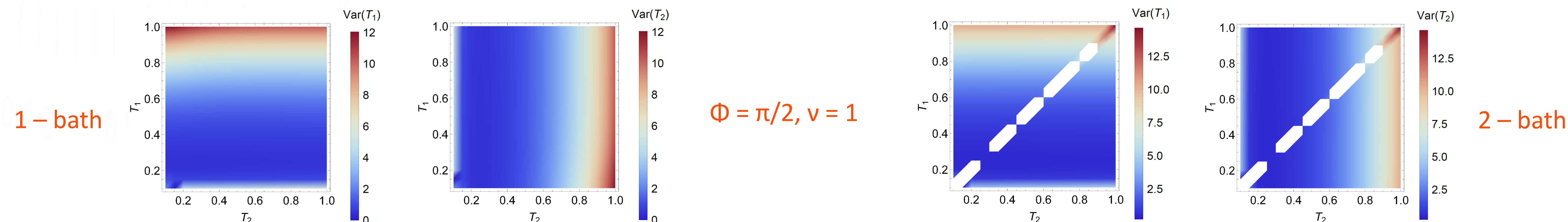
1 - bath case:



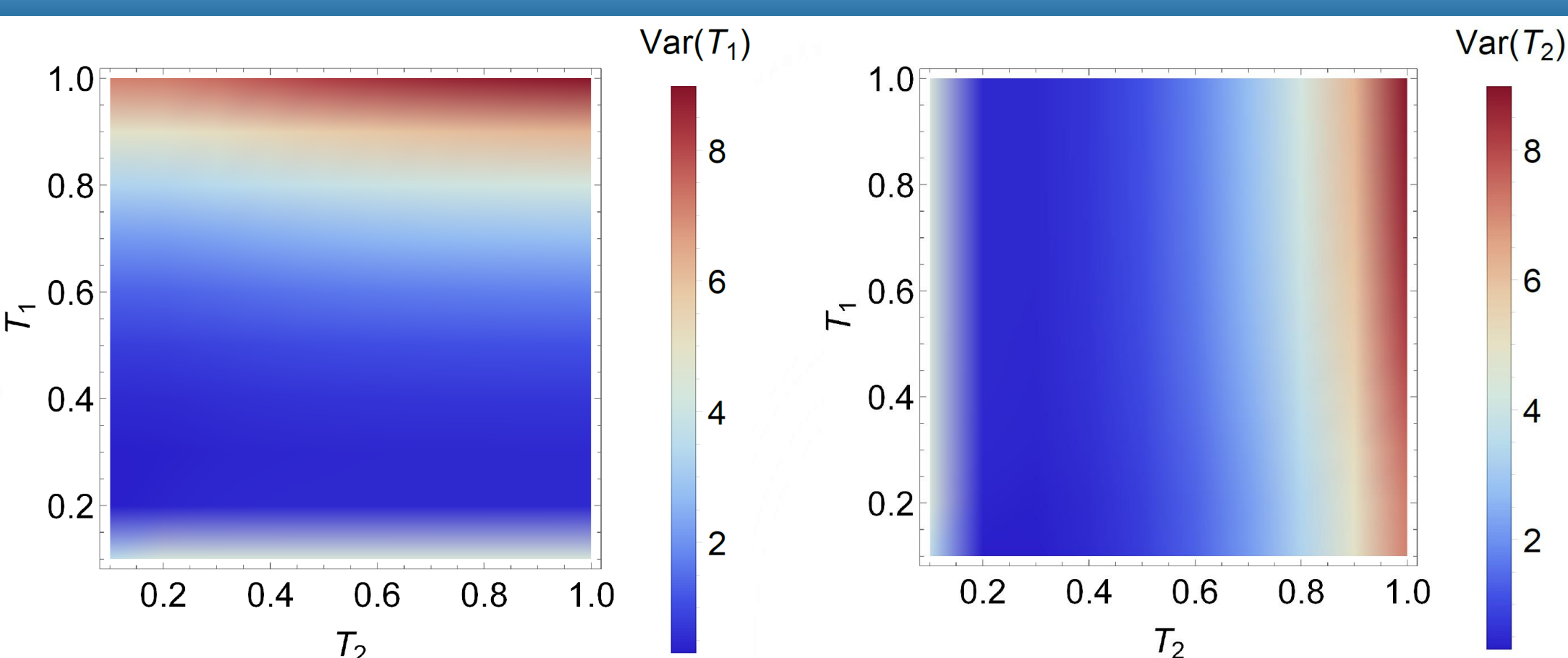
- Refined question: How effectively can we measure the temperature of the bath(s) in the above 2 approaches ?

NUMERICAL RESULTS FOR TEMPERATURE SUPERPOSITIONS

For a single qubit as a probe: 1-bath/ 2-bath : $|QFIM| = 0 \Rightarrow$ Bounds cannot be obtained. For a two-qubit probe, $|QFIM| \neq 0$ (generally).



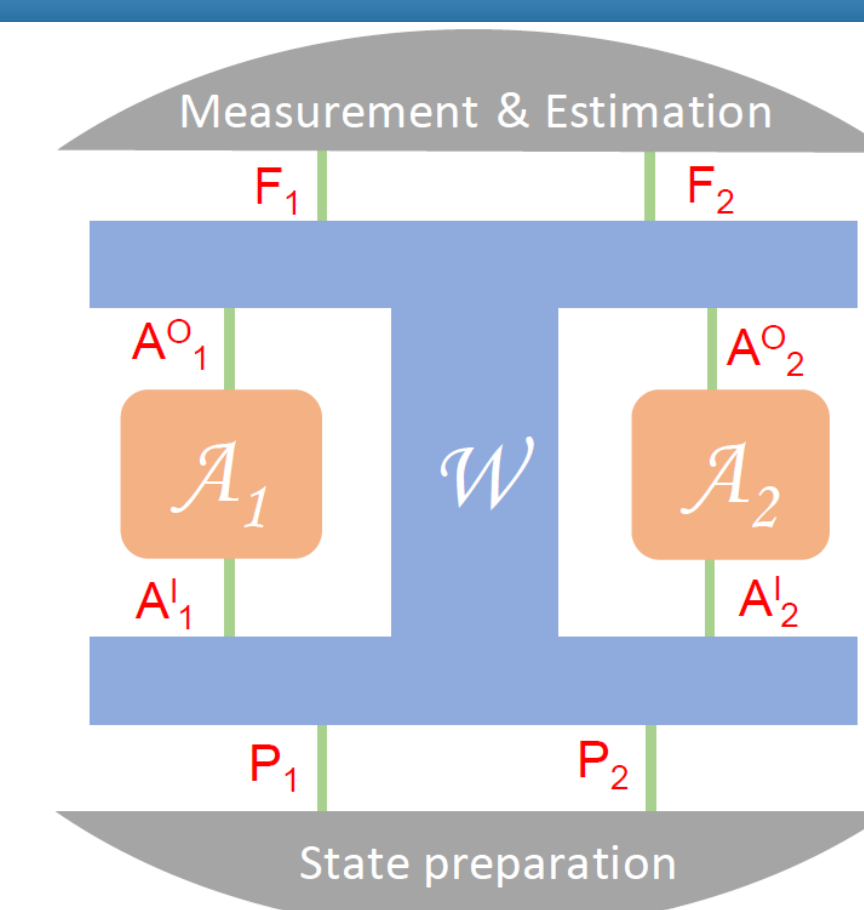
NUMERICAL RESULTS FOR THE QUANTUM SWITCH BASED SETUP



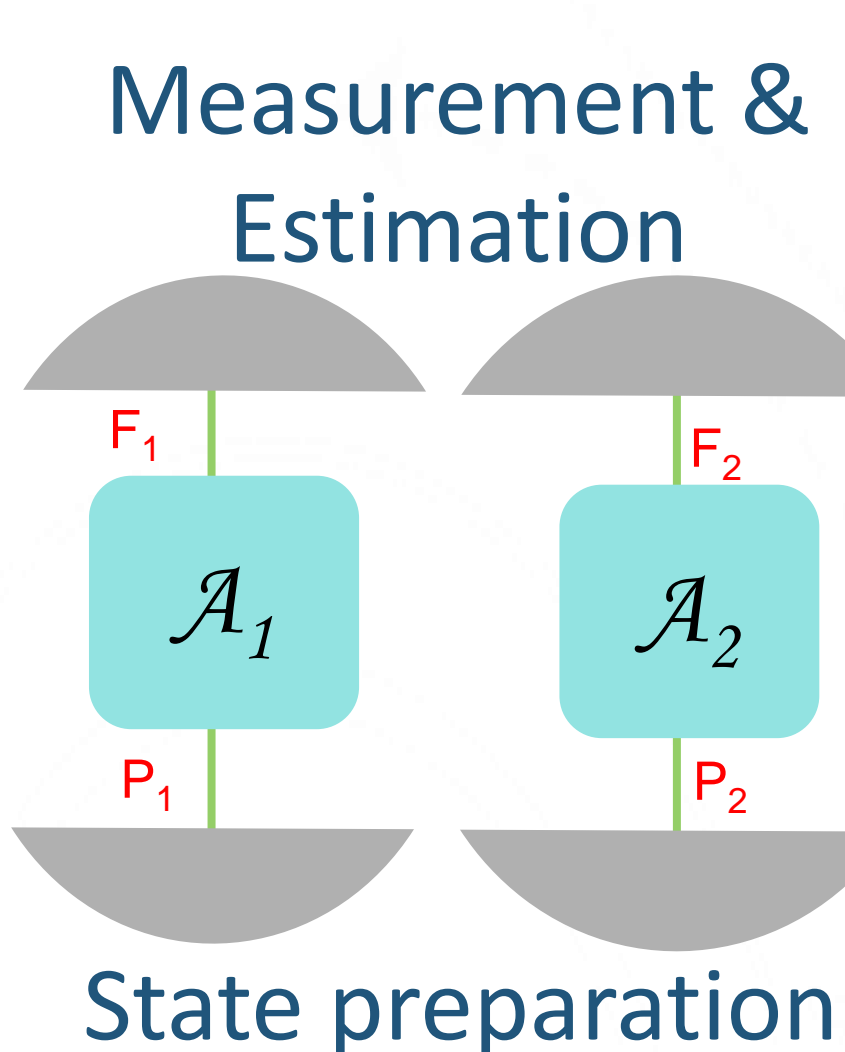
- Indefinite causal order via. Q - Ctrl over order of application of 2 thermalizing channels.

$$\rho_{\text{in}} = |0\rangle\langle 0| \otimes |+\rangle\langle +| \quad \nu = 1$$

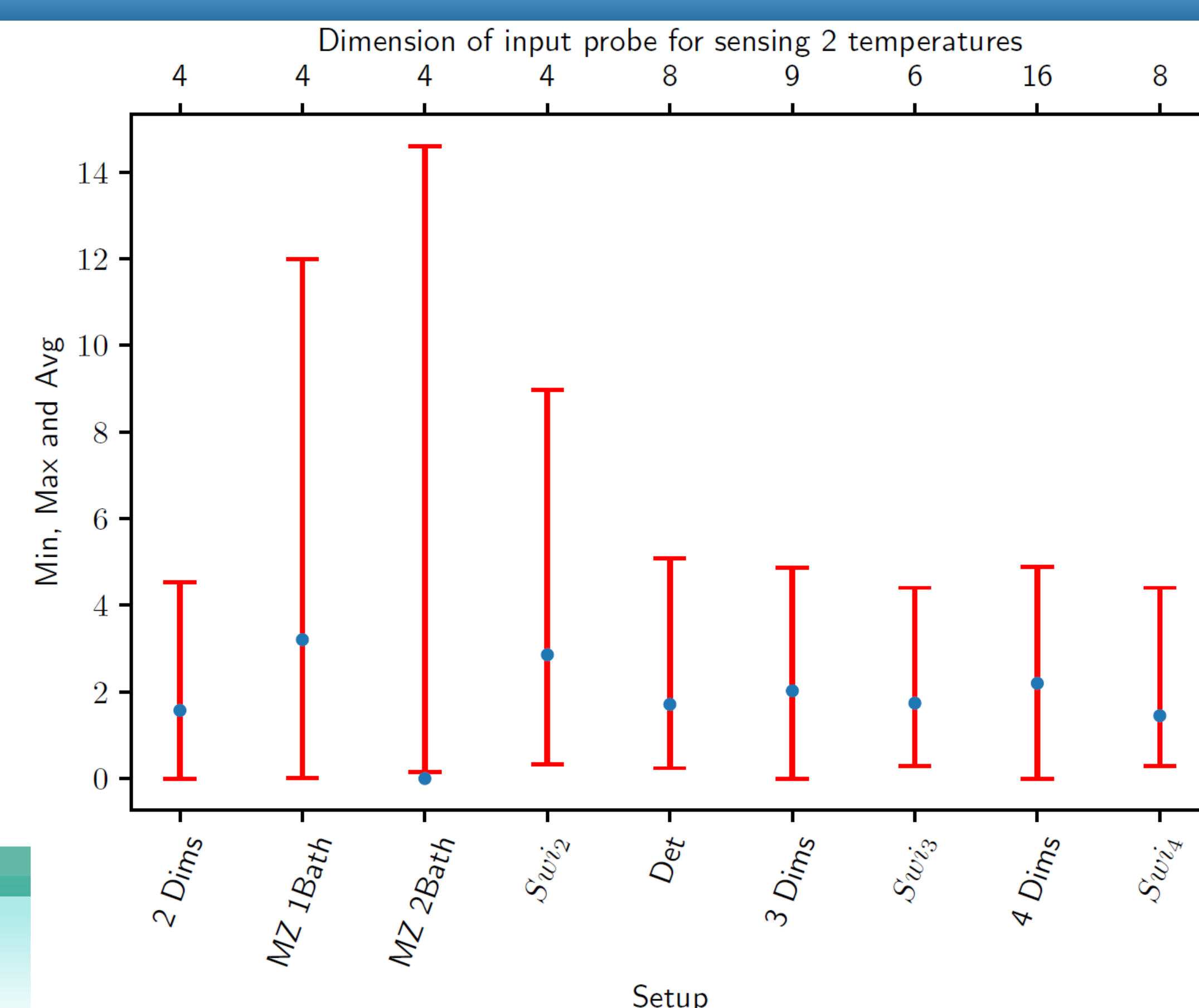
$\mathcal{A}_1, \mathcal{A}_2$: Quantum channels thermalizing to distinct temperatures.



CONCLUSIONS



- MZI can be used for the task of estimating 2 temperatures, with 2 qubits as a probe. However, the bounds are susceptible to Φ .
- Setups based on quantum switch, and other quantum processes are also useful for this task.
- Parallel scheme with simultaneous measurement of two temperatures using two independent probes usually outperforms all the others setups.
- However, if the dimension of the probe is increased, a quantum switch might outperform the parallel scheme.



REFERENCES

[1] Wood, C.E., Verma, H., Costa, F., & Zych, M. Operational models of temperature superpositions. arxiv 2112.07860
[3] Liu, J., Yuan, H., Lu, X.-M., & Wang, X. Quantum Fisher Information Matrix and Multiparameter Estimation. Journal of Physics A: Mathematical and Theoretical, 53(2), 23001 (2019).