Two-temperature estimation in the quantum regime: using Mach-Zehnder interferometer and quantum process framework

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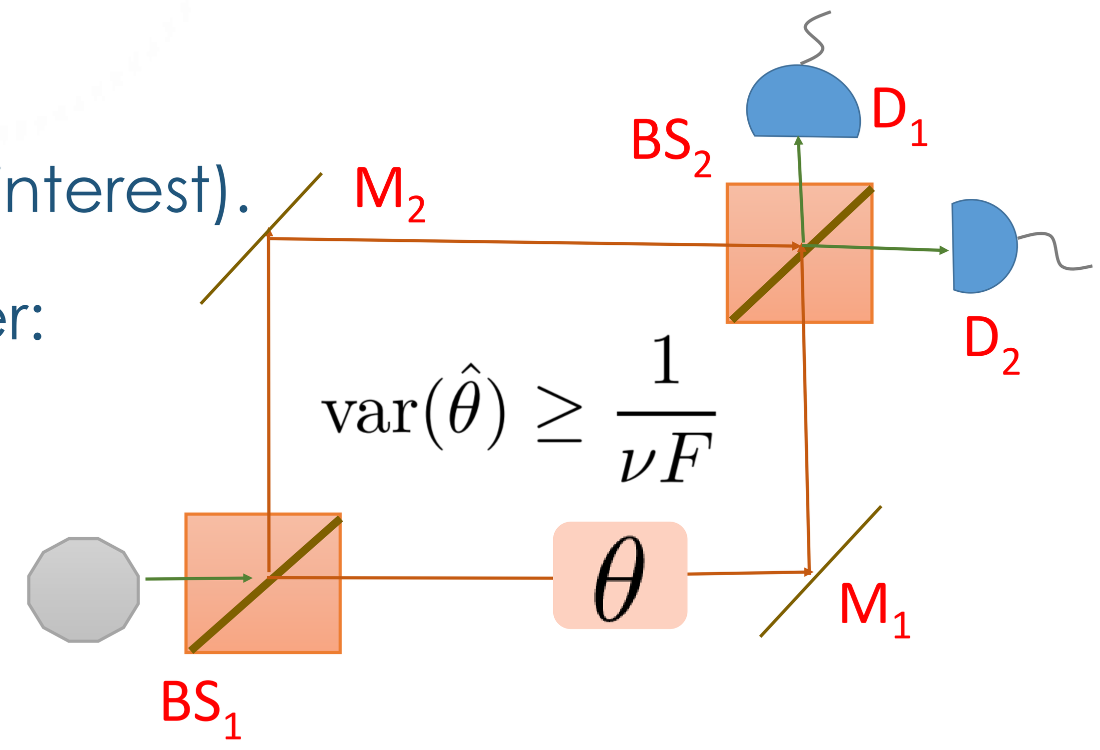
INTRODUCTION

- Question: What does it mean to have a superposition of thermalizing quantum channels ?
- Approach:
  - Exert Q-ctrl over the state of a bath – prepare it in a superposition of purified Gibbs' state.
  - Exert Q-ctrl over the path of the probe, prepare it in a superposition where it goes through 2 different baths.
- Refined question: How effectively can we measure the temperature of the bath(s) in the above 2 approaches ?

MULTIPARAMETER ESTIMATION & CR BOUND(S)

- How well can a parameter be estimated ?
- Density matrix – contains all the information we need (nonlinear fn. of parameter of interest).
- QFI: Mostly used in context of phase estimation – for e.g. in case of MZ interferometer:



$$\mathrm{var}(\hat{\theta}) \geq \frac{1}{\nu F}$$

ν – Repetitions of expt.
F – Q. Fisher Information

Our scheme : State preparation of qubit(s) as probe(s) → Temperature(s) encoded through interaction with bath(s) → Measurement and estimation of temperature

THERMALIZATION MODEL

Purified thermal bath constituted by 2 qubits:

$$|\theta^\beta\rangle \equiv \frac{e^{-E_0\beta/2}}{\sqrt{e^{-E_0\beta}+e^{-E_1\beta}}}|0,0\rangle + \frac{e^{-E_1\beta/2}}{\sqrt{e^{-E_0\beta}+e^{-E_1\beta}}}|1,1\rangle$$

$$T = 1/\beta$$

$$H_B = H_0 \otimes \mathbb{I}_2 + \mathbb{I}_2 \otimes H_0$$
$$H_0 = E_0|0\rangle\langle 0| + E_1|1\rangle\langle 1|$$

Interaction unitary (U):

$$U^\eta_{PB} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{1-\eta} & \sqrt{\eta} & 0 \\ 0 & -\sqrt{\eta} & \sqrt{1-\eta} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Multiparameter CR bound :

$$\mathrm{Cov}(\vec{T}) \geq \frac{\mathcal{Q}_T^{-1}}{\nu}$$

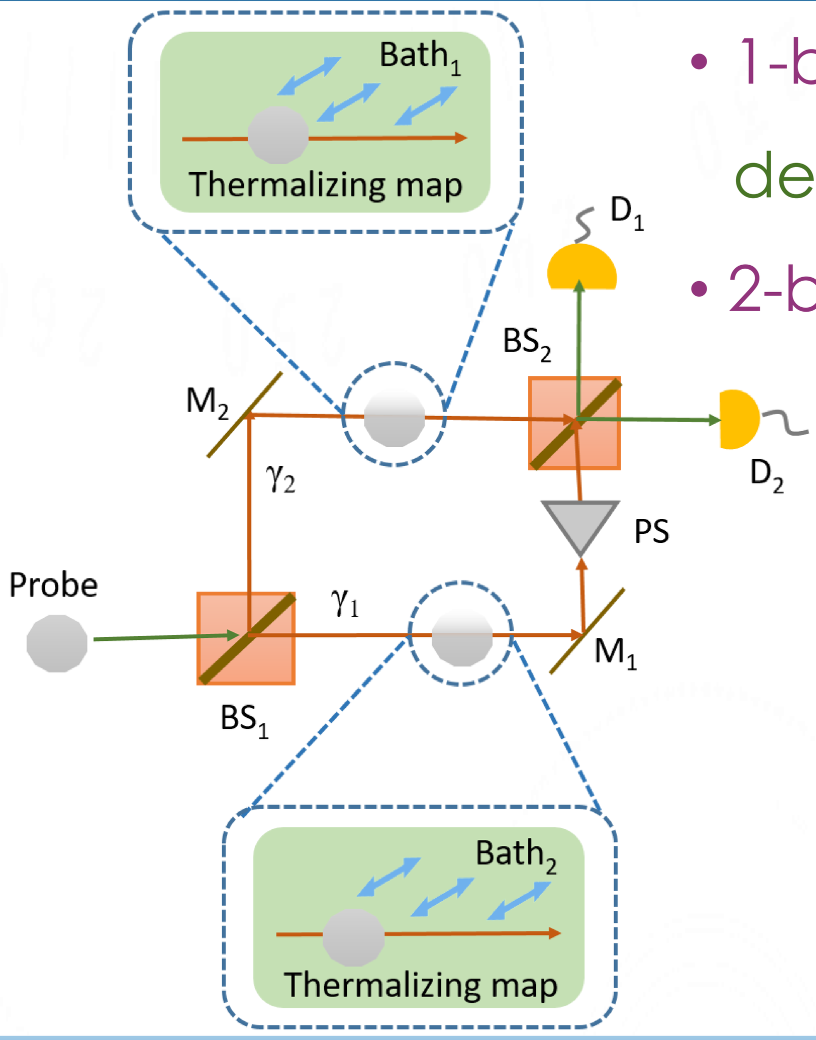
Bounds on variances obtained from the above matrix inequality :

$$\mathrm{Var}(T_1) \geq \frac{\mathcal{Q}_{T_2T_2}}{\nu\det(\mathcal{Q}_\mathcal{T})}$$

$$\mathrm{Var}(T_2) \geq \frac{\mathcal{Q}_{T_1T_1}}{\nu\det(\mathcal{Q}_\mathcal{T})}$$

$$\left(\mathrm{Var}(T_1) - \frac{\mathcal{Q}_{T_2T_2}}{\nu\ \det(\mathcal{Q}_\mathcal{T})}\right)\left(\mathrm{Var}(T_2) - \frac{\mathcal{Q}_{T_1T_1}}{\nu\det(\mathcal{Q}_\mathcal{T})}\right) \geq \left[\mathrm{Cov}(T_1,T_2) + \frac{\mathcal{Q}_{T_1T_2}}{\nu\det(\mathcal{Q}_\mathcal{T})}\right]^2$$

NUMERICAL RESULTS FOR THE MACH ZEHNDER SETUP

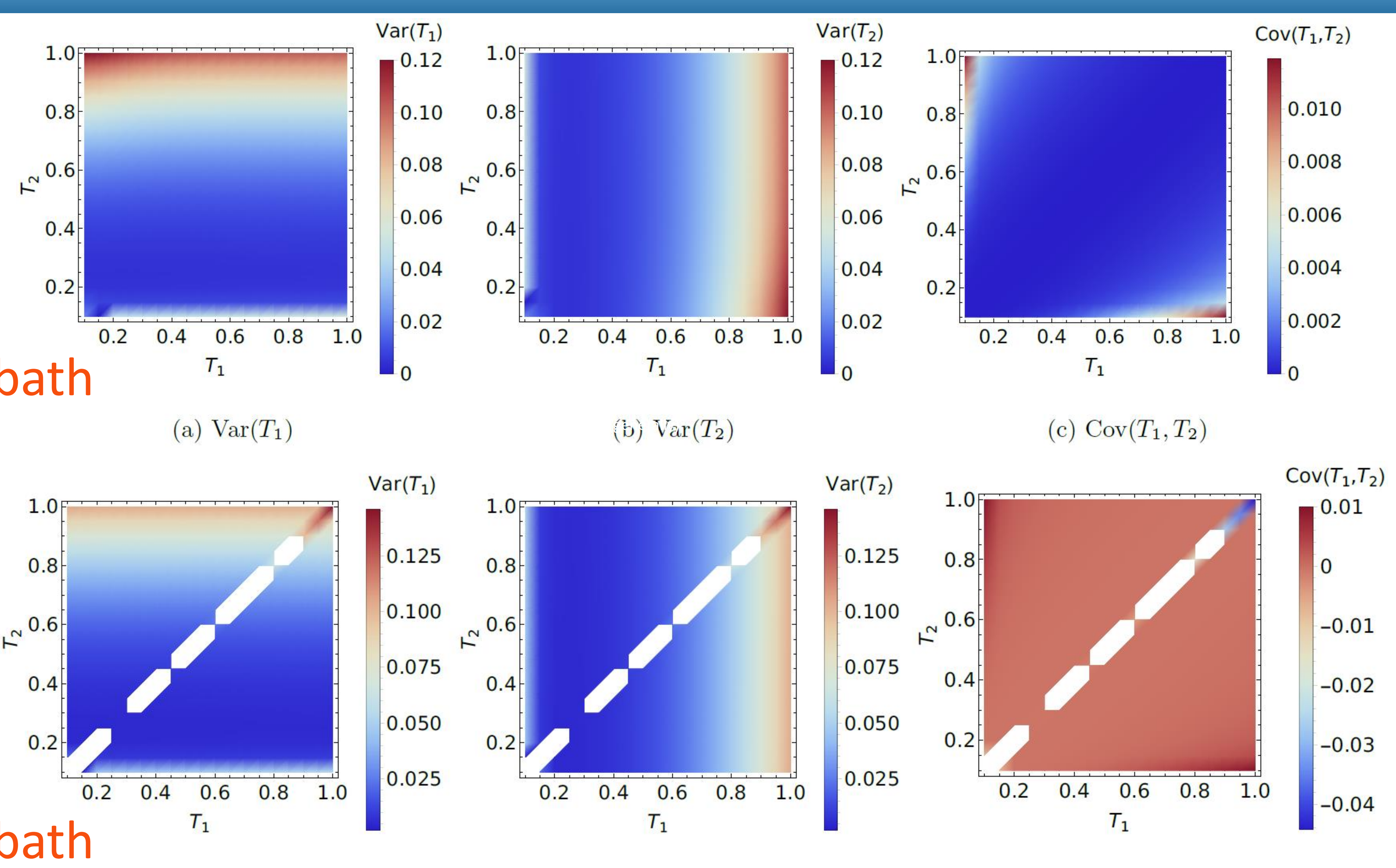


- 1-bath: The state of the (thermal) bath depends on the control (path).
- 2-bath: Two independent baths on the paths.

For a single qubit as a probe:

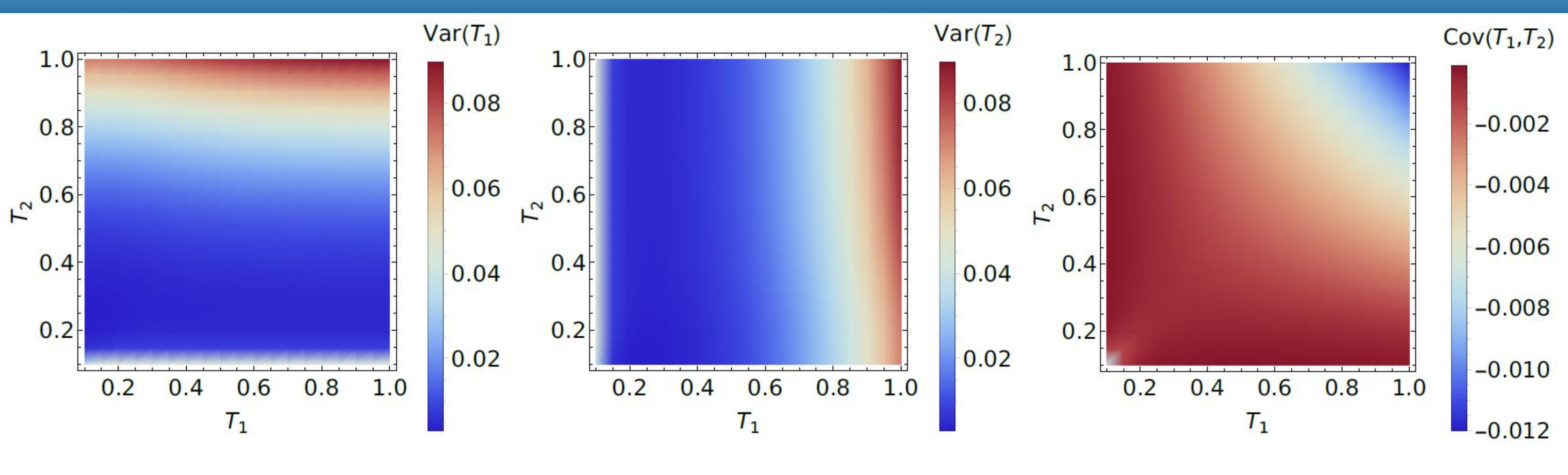
- 1-bath/ 2-bath : |QFIM| = 0 ⇒ Bounds cannot be obtained.
- For a two- qubit probe, |QFIM| ≠ 0 (generally).

$\rho_{\text{in}} = |0\rangle\langle 0| \otimes |0\rangle\langle 0|$ Φ = π/2, ν = 100



(a) Var(T_1) (b) Var(T_2) (c) Cov(T_1, T_2)

NUMERICAL RESULTS FOR THE QUANTUM SWITCH BASED SETUP

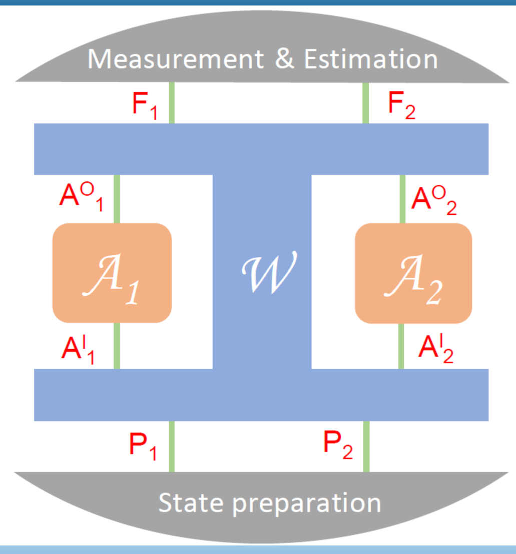


(a) Var(T_1) (b) Var(T_2) (c) Cov(T_1, T_2)

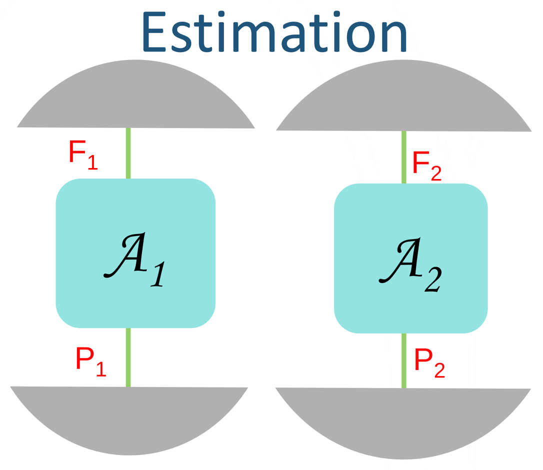
- Indefinite causal order via. Q - Ctrl over order of application of 2 thermalizing channels.

$\rho_{\text{in}} = |0\rangle\langle 0| \otimes |+\rangle\langle +|$ ν = 100

$\mathcal{A}_1$, $\mathcal{A}_2$: Quantum channels thermalizing to distinct temperatures.



CONCLUSIONS



- Mach-Zehnder interferometer can be used for the task of estimating 2 temperatures, with 2 qubits as a probe.
- Qualitatively similar results for setups based on other quantum processes.
- Parallel scheme with simultaneous measurement of two temperatures using two independent probes outperforms all the others.

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